Fermilab

TM-891
1620

SUPPORT LOCATION FOR ED DIPOLE MAGNET

W.B. Hanson and M. Leininger

August 2, 1979

The purpose of this investigation is to determine the best location for the support of the Energy Doubler dipole magnets. Ideally the supports would be placed so that the sag at the center is equal to that at the ends. However when the supports are considerably far from the ends, the end location tends to fluctuate from magnet to magnet based on measured data. From the standpoint of predictability of the end location for connection purposes and beam location it seems to be desirable to favor the ends by moving the supports toward the ends. In the limit, of course, the supports would be at the ends and the deflection at those points would be zero but the deflection at the center would be .180". The minimum calculated deflection location for the supports for 21' magnets is 130" apart with the calculated deflection to be .004" at the ends and center. (See attached calculations)

The 22' magnets were placed close to the ideal location, or $136\frac{3}{4}$. Inspection data revealed, however, that the end location fluctuated greatly and more often than not the sag at the ends was more than at the center. When the magnet was changed from 22' to 21' we made a judgement based upon the experience with the 22' magnets and chose a support spacing of 142" (12" greater than the calculated ideal).

Recently four 21' magnets were selected for a careful measurement of their deflections - magnets 207, 210, 211 and 213. The magnets were measured in the normal fashion taking measurements at the magnet cross-section center line at 4' increments along the length starting at the center plus a measurement at the last available measurement point at 114" from the center (12" from straight section ℄). In order to eliminate any built-in deviation from being a straight magnet to begin with, the measurements were made with the magnet set in the normal position and then repeated with the magnet turned over. The average of each reading then represents the actual deflection curve of the magnet. (The magnet support consisted of a single point at one end and two points at the other to prevent twisting. Also two measurements were taken at each longitudinal position and the average taken, again to eliminate the effect of any built-in twist).

Attached here-to are the graphs showing the deflection curves of each of these magnets. The center deflection varies from .006" to .017" with an average .009. The end deflections vary from a sag downward of .013 to an upward deflection of .002 with the average being .005 sag downward. However, if the last point measured is extrapolated to the straight section ℄ the average value for four magnets measured is .014.

Therefore, the conclusion is that 142" support spacing is not bad but an increased spacing would be preferred. 150" spacing would reduce the end deflection to .010" and increase the center to .016. Remember that the fluctuation of the ends has proven to be greater than the center and this change would improve this aspect also.

Other conclusions and recommendations:

1. The presently produced yokes are not straight enough to satisfy the criteria that they shall be straight within a .030" envelope. The half-yoke stacking fixture, the stacking assembly procedure, and the sagitta assemble table should be checked out. (A new improved sagitta table is in process).

2. The yoke straightness varies from yoke to yoke excessively. The weld technique should be changed to improve uniformity of penetration from intermittent MIG welds to continuous TIG welds.

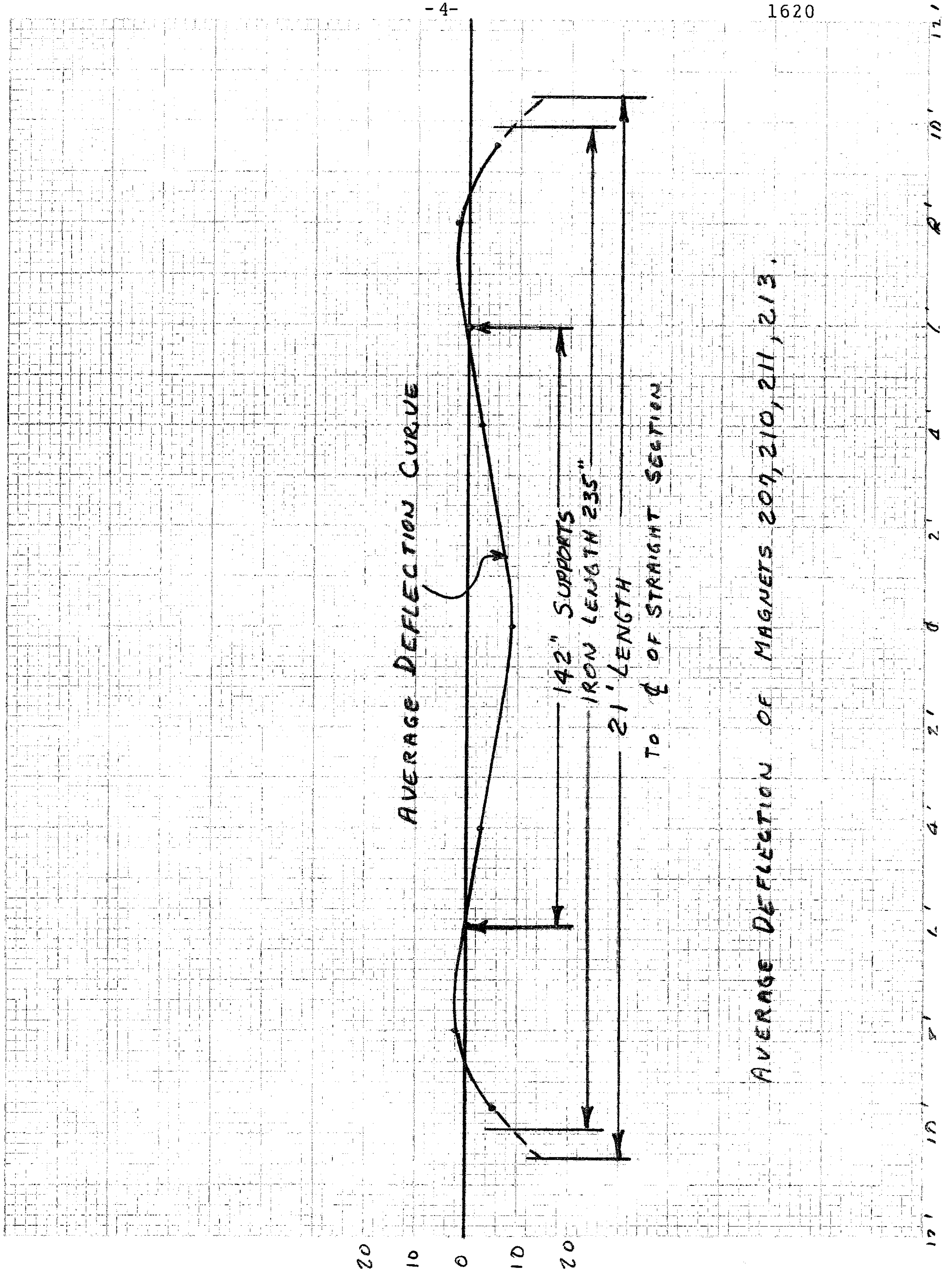
3. The impregnation of the yokes with epoxy could be improved by applying a high pressure to the epoxy at both ends after filling - say 100 psi. Also the lamination cleaning technique could be improved. A vacuum impregnation of the yokes would even be superior but this would be considerably more complicated.

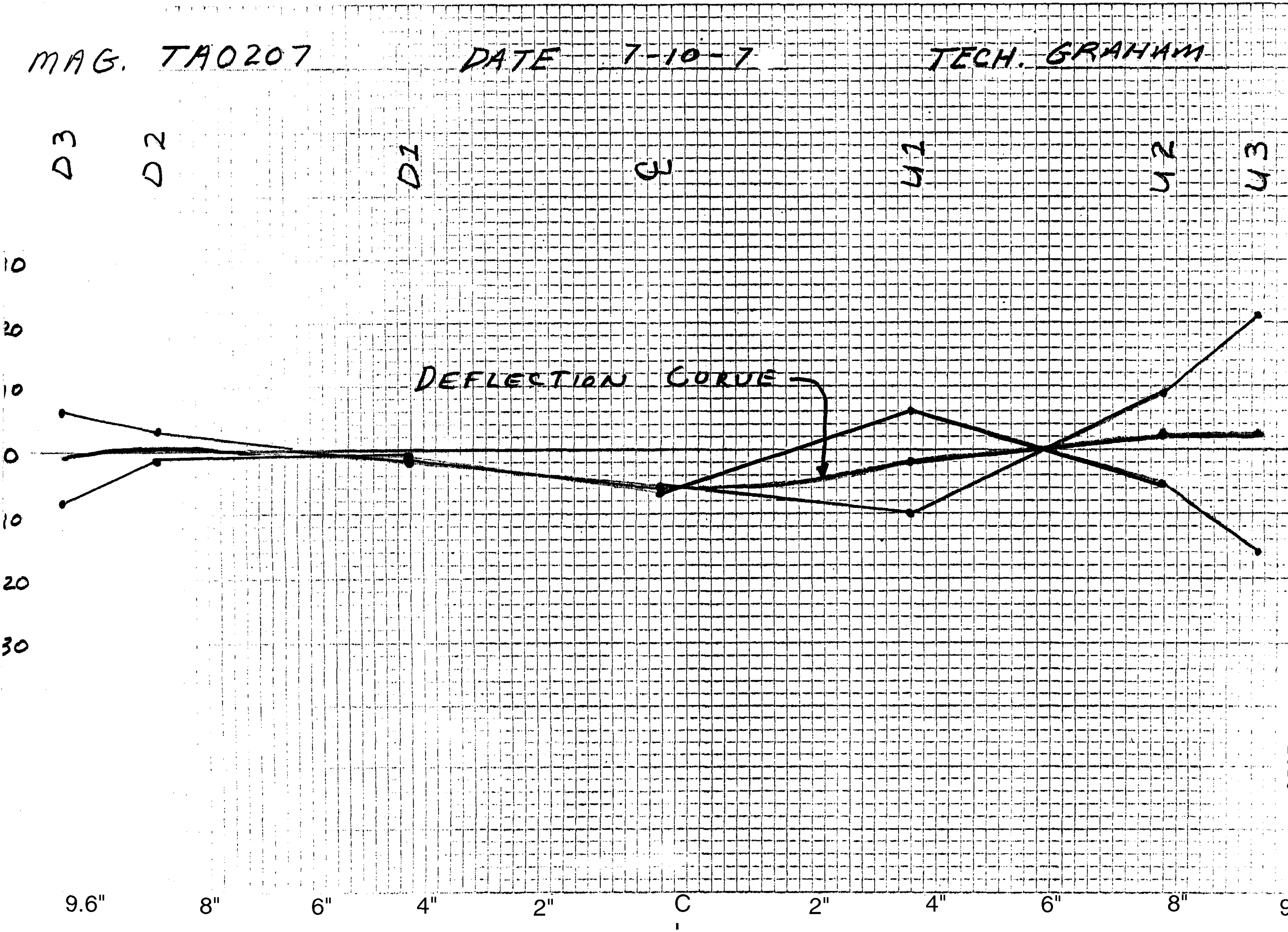
4. Four additional magnets should be completely measured including the deflection at the very ends of the magnet and at 2' increments.

5. The empirical formulae used for calculating the deflections and inertia moments are apparently incorrect for this application since they give results so far from the actual measured values.

Addendum

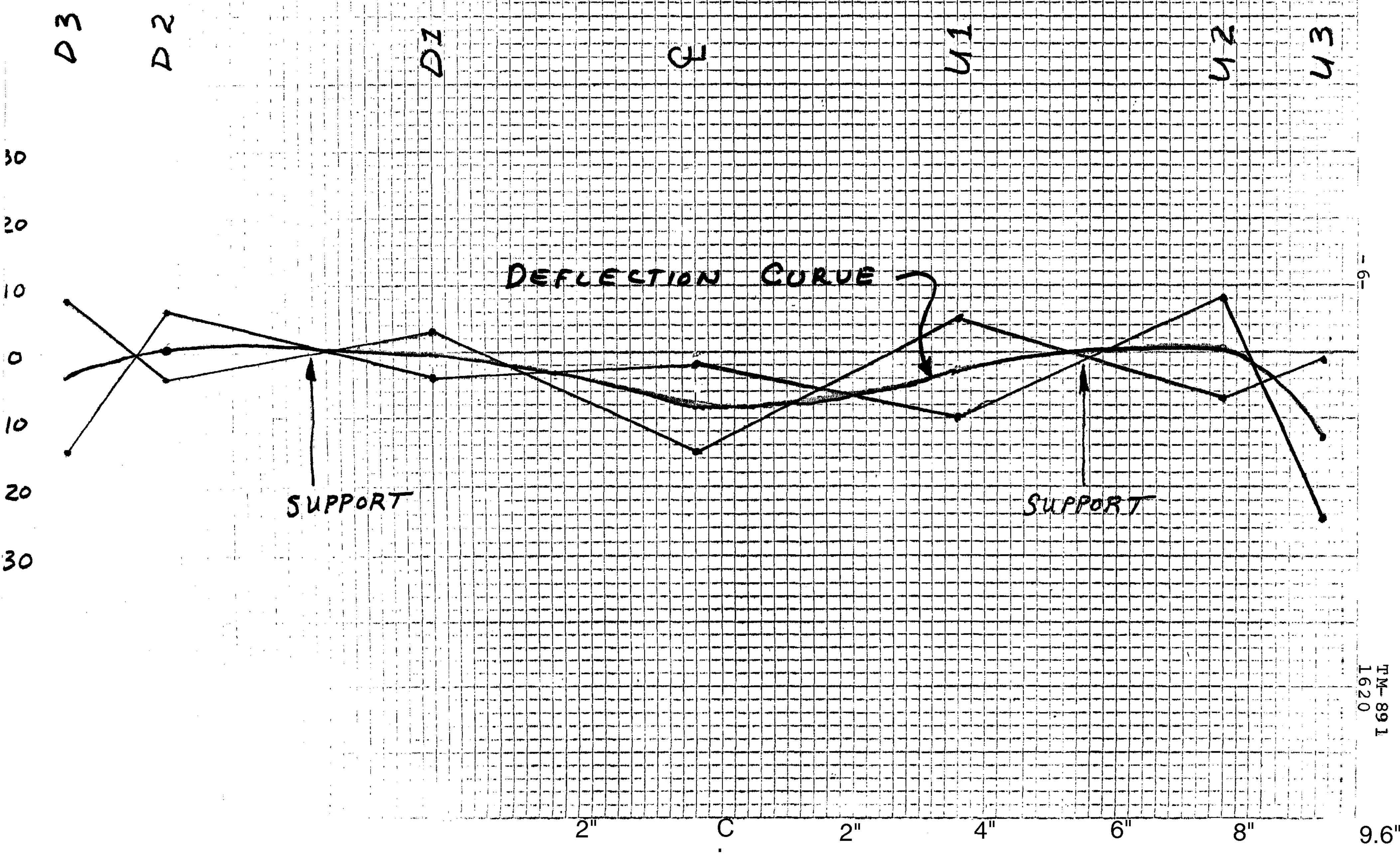
R. Shafer has also calculated these deflections using a different approach but he gets essentially the same resuilts. His calculations are also attached for completeness.


AVERAGE DEFLECTION CURVE
142" SUPPORTS
IRON LENGTH 235"
21' LENGTH
TO ℄ OF STRAIGHT SECTION
AVERAGE DEFLECTION OF MAGNETS 207, 210, 211, 213.
20
10
0
10
20

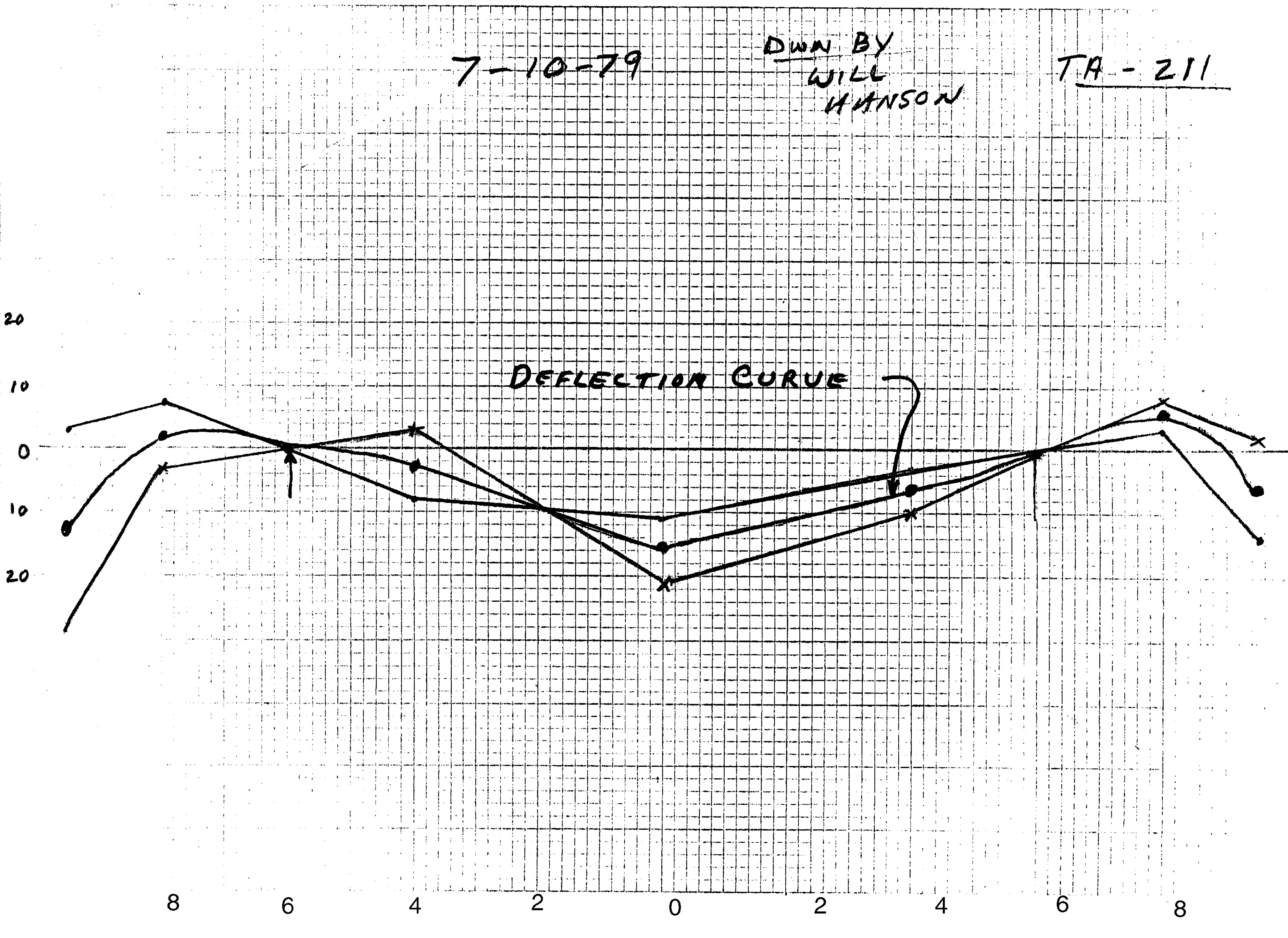

MAG. TA0207
DATE 7-10-7
TECH. GRAHAM
D3
D2
D1
℄
U1
U2
U3
DEFLECTION CURVE
10
20
10
0
10
20
30
9.6"
8"
6"
4"
2"
C
2"
4"
6"
8"
9.6"

TM-891
1620

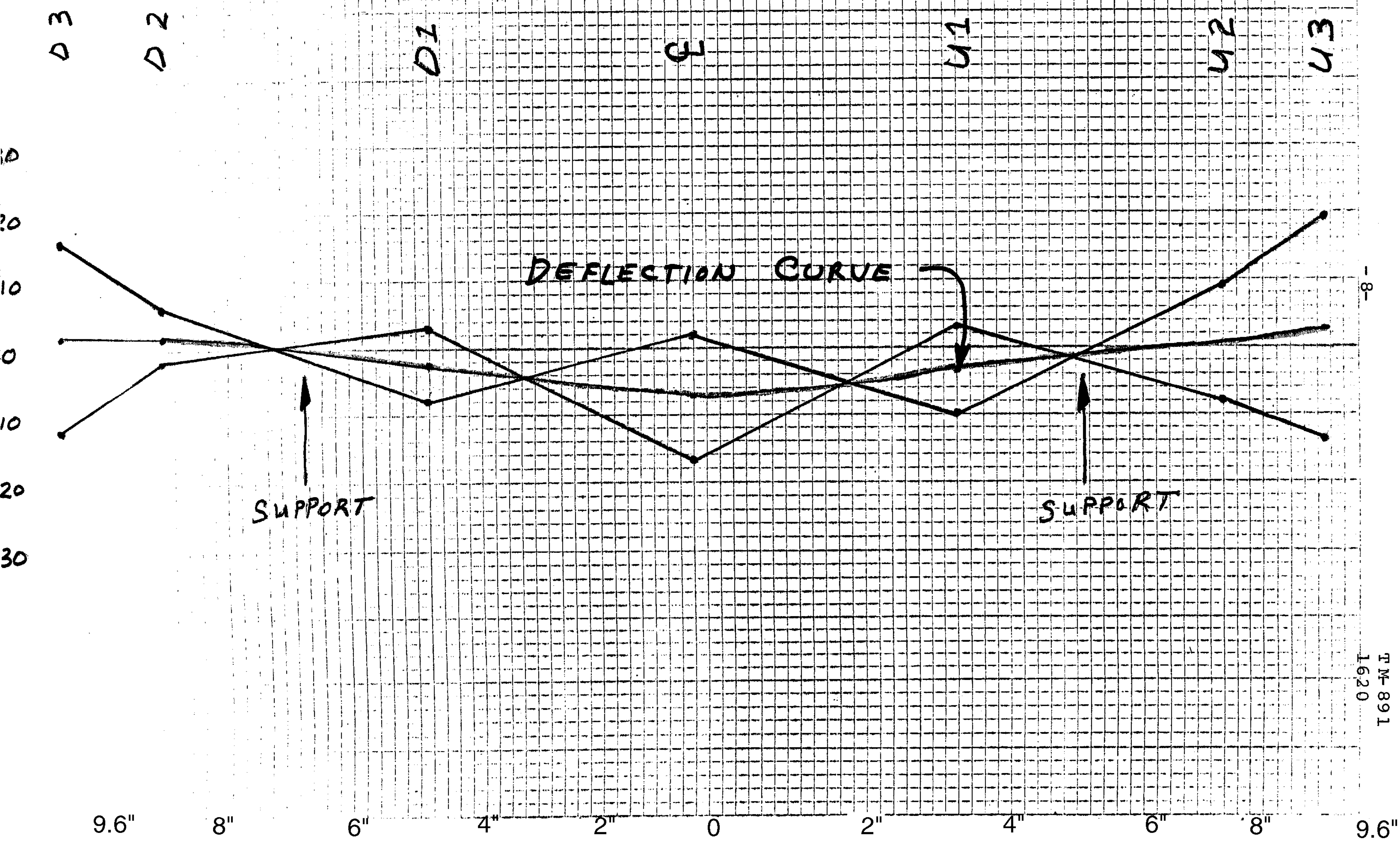
MAG TA0210 DATE 7-10-79 TECH. GRAHAM



TM-891
1620


7-10-79
DWN BY
WILL
HANSON
TA - 211
DEFLECTION CURVE
20
10
0
10
20
8
6
4
2
0
2
4
6
8

MAG TA 0213 DATE 7-10-79 TECH. GRAHAM



TM-891
1620

SUMMARY

FOR A CHANGE IN SUPPORT SPACING ΔS (POSITIVE CHANGE MEANS WIDER SPACING) THE FOLLOWING CHANGES IN DEFLECTIONS TAKE PLACE (POSITIVE DEFLECTION IS DOWN, NEGATIVE IS UP): IN INCHES

Δ CENTER DEFLECTION = $(9.26E-4)\ \Delta S$ CENTER

Δ END DEFLECTION = $-(6.63E-4)\ \Delta S$ END

NOTICE THAT AS SPACING INCREASES THE CENTER SAGS BUT THE ENDS RISE AS INDICATED BY THE NEGATIVE SIGN.

THE THEORETICAL IDEAL DEFLECTION (SAME AT CENTER AND END) WOULD BE .0037 in.

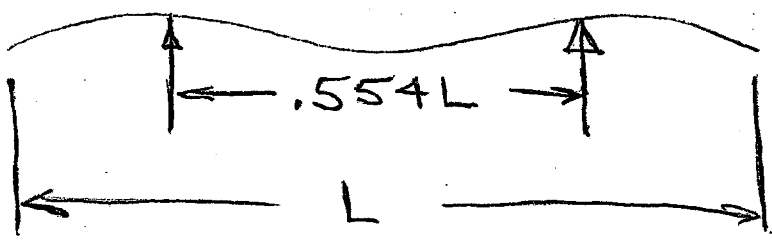
MARK
7·11·79

21' MAGNET SUPPORT SPACING

PROBLEM: WHAT SUPPORT SPACING IS REQUIRED TO CAUSE EQUAL DEFLECTION AT CENTER AND ENDS?

GIVEN MAGNET IRON 235" $EI = 7.87E9$ psi
$wL = W = 8400$ lb

SOLUTION



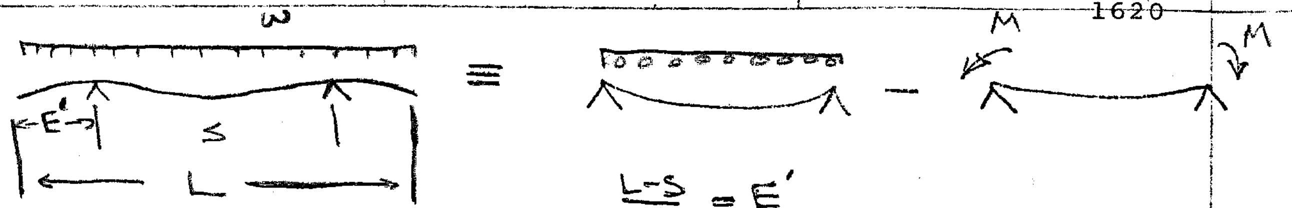
SUPPORT SPACING SHOULD BE

$$S = (.554)(235") = 130"$$ ← THEORETICAL SUPPORT SPACING

THE DEFLECTION IN THIS CASE IS

$$\Delta = .000268 \frac{wL^4}{EI} = .0037 \text{ in}$$

NEXT PROBLEM BASED ON WHAT WAS LEARNED FROM SUPPORT SPACING OF 22' MAGNET AND ITS CORRESPONDING SHAPE, AN ATTEMPT TO MODIFY 21' SUPPORT SPACING TO ACHIEVE EQUAL END AND CENTER DEFLECTIONS WAS MADE. BY SCALING THE PREVIOUS ERROR AND NECESSARY CORRECTION. THE SPACING TRIED WAS 142". THIS SPACING CAME CLOSE BUT WAS STILL INCORRECT. THE NEXT CORRECTION WILL BE MADE BY FINDING THE INCREMENT IN DEFLECTION FOR CHANGE IN SUPPORT SPACING AROUND 142"



$$\frac{L-S}{2} = E'$$

$$\delta_{center} = \frac{5}{384}\frac{w\cancel{L}S^4}{EI} - (2)(.0625)\frac{MS^2}{EI} \quad \text{and } M = \frac{\frac{w(L-S)}{2}\cdot E'}{2} = \frac{wE'^2}{2}$$

L^4

$$\delta_{center} = \frac{5}{384}\frac{wLS^3}{EI} - (2)(.0642)\frac{w(E'S)^2}{2EI}$$

$$\theta = \frac{1}{24}\frac{wLS^2}{EI} - \frac{1}{2}\frac{wE^2S}{2EI} = \frac{1}{24}\frac{wS^3}{EI} - \frac{1}{4}\frac{wE'^2S}{EI}$$

Find change in δ_{center} with respect ~~to~~ S:

change $\xrightarrow{}$ LS^3

$$\frac{d(\delta_{center})}{dS} = \frac{15}{384}\frac{wLS^2}{EI} - \frac{w(2)(.0642)(E'S)}{EI}\left[\frac{dE'}{dS}\cdot S + E'\right]$$

now $\frac{dE'}{dS} = -\frac{1}{2}$ so

$$\frac{d(\delta_{center})}{dS} = \frac{15}{384}\frac{wLS^2}{EI} - \frac{w(2)(.0642)(E'S)}{EI}\left[-\frac{S}{2} + \frac{L-S}{2}\right]$$

$$= \frac{15}{384}\frac{wLS^2}{EI} + \frac{(.0642)(L-S)(S)^2 w}{2EI} - .0642\frac{(L-S)^2 S w}{2EI}$$

$$\frac{WL^3L^2}{LWL^4} = [0] \qquad \frac{L^3L^2}{WL^4}\frac{w}{L} = [0] \qquad \frac{L^3WL^2}{LWL^4} = 0$$

$$\frac{EI}{w}\frac{d(\delta_{center})}{dS} = \frac{15LS^2}{384} + .0321(L-S)S^2 - .0321(L-S)^2S$$

$$= .0391LS^2 + .0321(L)S^2 - .0321S^3 - .0321(L^2S)$$

$$+ .0642LS^2 - .0321S^3$$

$$= -.0642S^3 + .135LS^2 - .0321L^2S$$

FOR A SPACING OF 142" (S=142) THIS BECOMES

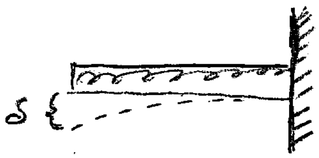
$$\frac{EI}{w}\frac{d\delta_{center}}{dS} = 2.04E5 \text{ in}^3$$

So the change in deflection in the center can be approximated by

$$\Delta\delta_{center} = \frac{w}{EI}(2.04E5 \text{ in}^3)\Delta S$$ ← Change in center deflection with respect to S

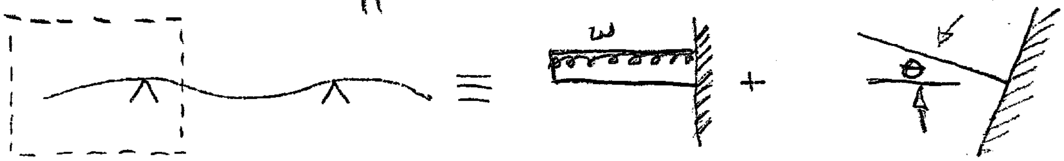
NEXT FIND VARIATION in end deflection with respect to S:

TREAT END PIECE AS CANTILEVER WHICH GETS ROTATED θ (calculated on second page)



$$\delta = 1/8 \frac{wE'^4}{EI}$$

The actual deflection of the end of magnet is reduced because of the rotation at the support:



where θ was calculated on page 2 as:

$$\theta = 1/24 \frac{wLS^2}{EI} - 1/4 \frac{wE'^2 S}{EI} \qquad \frac{WL^3}{LWL^4} L^2 = [0], \quad \frac{WL^2 L^2 L}{LWL^4} = [E$$

The deflection due to rotation is $\delta_\theta = E'\theta$

$$\delta_{end} = \delta - \delta_\theta = 1/8 \frac{wE'^4}{EI} - 1/24 \frac{wE'S^2}{EI} + 1/4 \frac{wE'^3 S}{EI}$$

$$\delta_{end} = 1/8 \frac{w}{EI}\left(\frac{L-S}{2}\right)^4 - 1/24 \frac{w}{EI} LS^2\left(\frac{L-S}{2}\right) + \frac{w}{4EI} S\left(\frac{L-S}{2}\right)^3$$

$$\frac{EI}{w}\delta_{end} = \frac{(L-S)^4}{128} - \frac{LS^2(L-S)}{48} + \frac{S(L-S)^3}{32}$$

now find variation of δ_{end} with respect to S:

$$\frac{EI}{w}\frac{d(\delta_{end})}{dS} = \frac{-4(L-S)^3}{128} - \frac{2LS(L-S)}{48} + \frac{S^2L}{48} + \cancel{\frac{(L-S)^3}{32}} - \frac{3S(L-S)^2}{32}$$

$$= \quad \frac{-LS(L-S)}{24} + \frac{S^2L}{48} - \frac{3S(L-S)^2}{32}$$

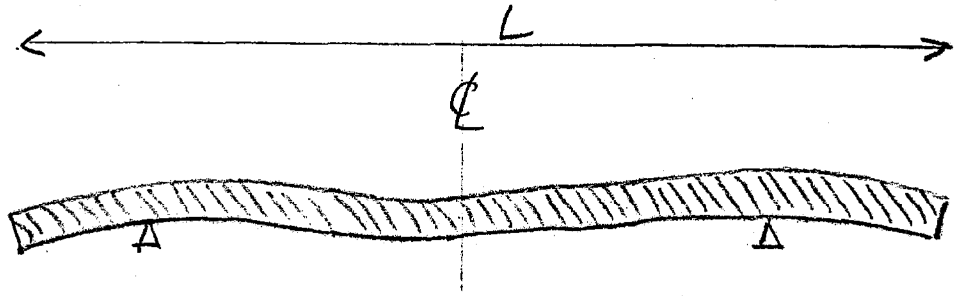
FOR $L = 235''$, $S = 142''$:

$$\frac{EI}{w}\frac{d(\delta_{end})}{dS} = -8.11 \text{ E4 in}^3$$

$$\Delta\delta_{end} = (-1.46\text{E}5 \text{ in}^3)\left(\frac{w}{EI}\right)(\Delta S) \longleftarrow$$ Change in end deflection with respect to S

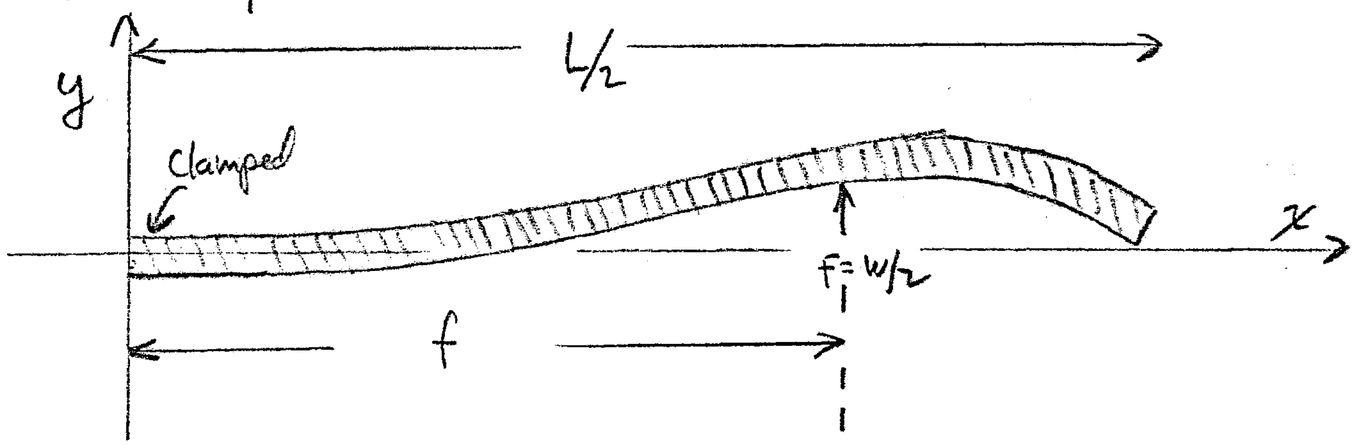
Calculation of the Deflection of FD Dipole Magnets on Symmetric Supports

We consider the deflection of a beam of length L and weight W uniformly distributed over its length, supported by two points equally distant from the center:



Due to symmetry about the center we may immediately simplify the problem to the following:

Consider a beam of length $L/2$ and weight $W/2$ (uniformly distributed over its length) clamped in a horizontal position at its left end with an upward vertical force $F = W/2$ at a distance $x = f$ from the clamped end:



General As the beam is static, the total stresses and torques acting on each element must be zero. We can therefore calculate the stress and torque at any point as follows:

$x < f$

Stress at x: $S = \frac{W}{2} - \int_x^{L/2} \frac{W}{L} dx' = \frac{Wx}{L}$

Torque at x: $N = \frac{W}{2}(f-x) - \int_x^{L/2} \frac{W}{L}(x'-x)dx'$

$$= \frac{Wf}{2} - \frac{WL}{8} - \frac{Wx^2}{2L}$$

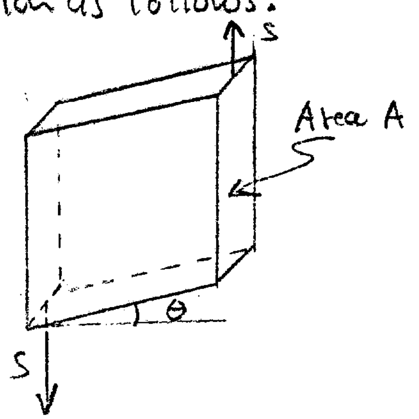
$x > f$

Stress at x: $S = -\int_x^{L/2} \frac{W}{L} dx' = \frac{W}{L}(x - \frac{L}{2})$

Torque at x: $N = -\int_x^{L/2} \frac{W}{L}(x'-x)dx'$

$$= -\frac{WL}{8} + \frac{Wx}{2} - \frac{Wx^2}{2L}$$

Shearing stress Shearing forces can cause a distortion of the shape of a section as follows:



$\theta = \frac{S}{nA}$ n = shear modulus

<u>Longitudinal stress</u> the relation between force per unit area and linear extension of a material is

$$\frac{dF}{dA} = E\frac{\Delta l}{l} = E z \frac{\Delta\varphi}{l} = E z \frac{d\varphi}{dx}$$

where E = Youngs modulus, z = distance up or down from neutral plane and $\frac{d\varphi}{dx}$ = curvature in neutral plane due to torque. We assume neutral plane passes through centroid of beam.

the total torque is then $N = \int z\,dF = E\frac{d\varphi}{dx}\int z^2 dA$

$$= EI\frac{d\varphi}{dx}$$

where I = moment of inertia about centroid

$$\therefore \frac{d\varphi}{dx} = \frac{N}{EI}$$

the slope of the beam at any point is given by

$$\frac{dy}{dx} = \varphi + \theta \qquad \text{(small angle approximation)}$$

$$\therefore \frac{d^2y}{dx^2} = \frac{d\varphi}{dx} + \frac{d\theta}{dx}$$

$$= \frac{N}{EI} + \frac{1}{nA}\frac{dS}{dx} = \frac{N}{EI} + \frac{w}{nAL}$$

$\underline{\underline{x < f}}$

$$\frac{d^2y}{dx^2} = \frac{1}{EI}\left[\frac{wf}{2} - \frac{wL}{8} - \frac{wx^2}{2L}\right] + \frac{w}{nAL}$$

$$\frac{dy}{dx} = \frac{1}{EI}\left[\frac{wfx}{2} - \frac{wLx}{8} - \frac{wx^3}{6L}\right] + \frac{wx}{nAL} + C_1$$

$$y = \frac{1}{EI}\left[\frac{wfx^2}{4} - \frac{wLx^2}{16} - \frac{wx^4}{24L}\right] + \frac{wx^2}{2nAL} + C_1 x + C_0$$

However, $C_1 = C_0 = 0$ since beam is clamped at $x=0$.

$\underline{\underline{x > f}}$

$$\frac{d^2y}{dx^2} = \frac{1}{EI}\left[-\frac{wL}{8} + \frac{wx}{2} - \frac{wx^2}{2L}\right] + \frac{w}{nAL}$$

$$\frac{dy}{dx} = \frac{1}{EI}\left[-\frac{wLx}{8} + \frac{wx^2}{4} - \frac{wx^3}{6L}\right] + \frac{wx}{nAL} + C_1'$$

$$y = \frac{1}{EI}\left[-\frac{wLx^2}{16} + \frac{wx^3}{12} - \frac{wx^4}{24L}\right] + \frac{wx^2}{2nAL} + C_1' x + C_0'$$

We must now match y and $\frac{dy}{dx}$ at $x=f$ to determine C_1' and C_0'

$$\left.\frac{dy}{dx}\right|_f : \quad \frac{1}{EI}\left[\frac{wf^2}{2} - \cancel{\frac{wLf}{8}} - \cancel{\frac{wf^3}{6L}}\right] = \frac{1}{EI}\left[\frac{wf^2}{4} - \cancel{\frac{wLf}{8}} - \cancel{\frac{wf^3}{6L}}\right] + C_1'$$

$$\therefore \quad C_1' = \frac{1}{EI}\frac{wf^2}{4}$$

$$y\Big|_f : \quad \frac{1}{EI}\left[\cancel{\frac{wf^3}{4}} - \cancel{\frac{wLf^2}{16}} - \cancel{\frac{wf^4}{24L}}\right] = \frac{1}{EI}\left[-\cancel{\frac{wLf^2}{16}} + \frac{wf^3}{12} - \cancel{\frac{wf^4}{24L}} + \cancel{\frac{wf^3}{4}}\right] + C_0'$$

$$\therefore \quad C_0' = -\frac{1}{EI}\frac{wf^3}{12}$$

We can also make the following simplification:

In general $n \sim \frac{E}{3}$ (Poisson ratio)

$$\therefore \quad \frac{wx^2}{2nAL} = \frac{3wx^2}{2EAL} = \frac{1}{EI}\frac{3Iwx^2}{2AL}$$

So we now have the following equations

$$x<f \qquad y=\frac{W}{EI}\left[\frac{fx^2}{4}+\frac{3Ix^2}{2AL}-\frac{Lx^2}{16}-\frac{x^4}{24L}\right]$$

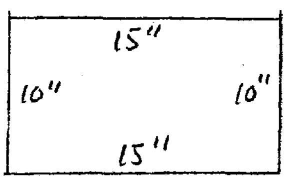
$$x>f \qquad y=\frac{W}{EI}\left[-\frac{f^3}{12}+\frac{f^2x}{4}+\frac{3Ix^2}{2AL}-\frac{Lx^2}{16}+\frac{x^3}{12}-\frac{x^4}{24L}\right]$$

where W = total weight
L = total length
E = Yangs modulus
I = moment of inertia of cross section thru centroid
A = cross sectional area
x = distance from center
y = vertical deflection at x assuming $y=0$ at $x=0$
f = distance of support from center.

We now consider an E.D. Dipole:

$W = 8400$ lbs ; $L = 235''$; $EI = 7.87\times10^9$ lb-in^2

cross section is



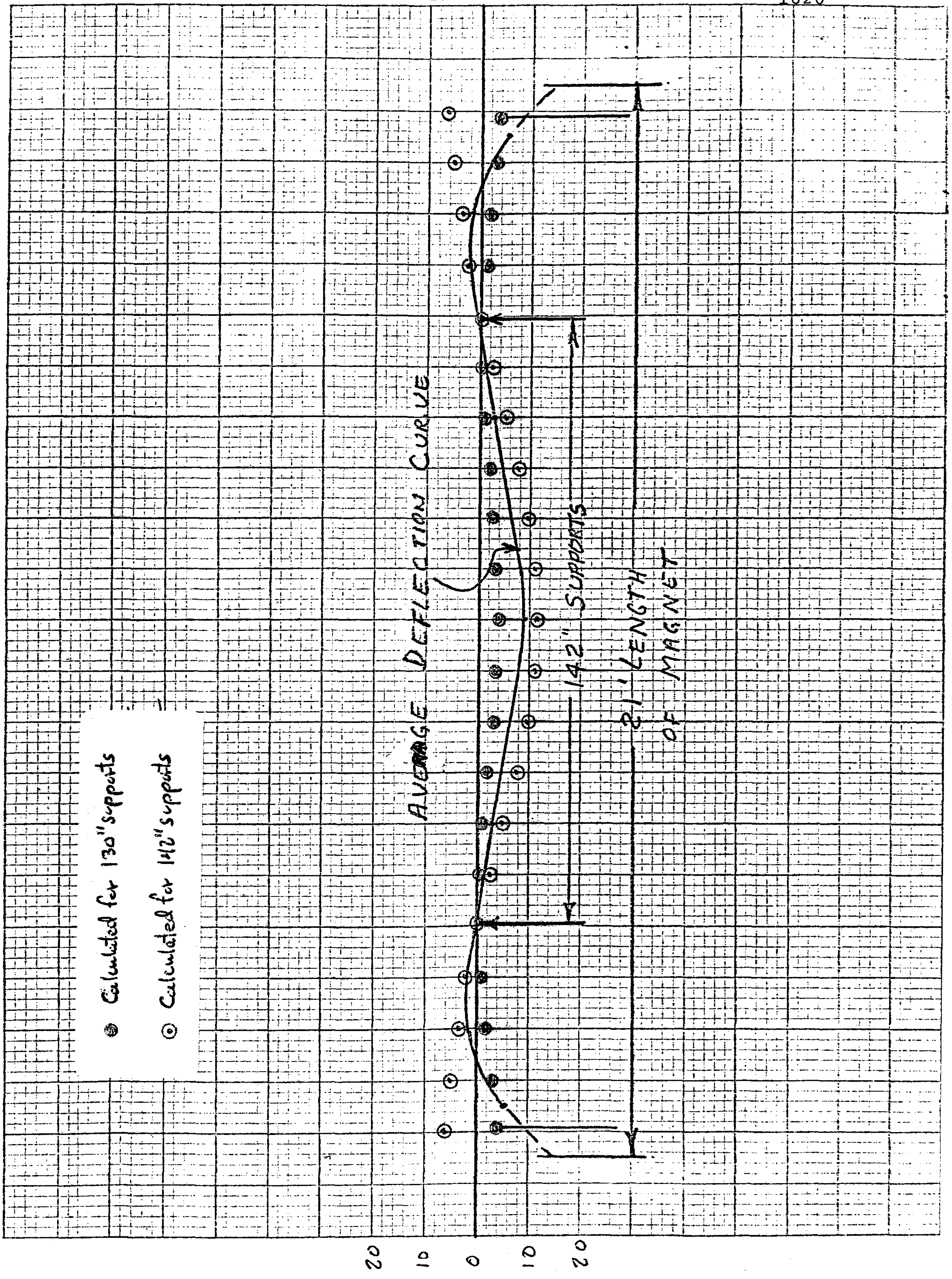
so $A = 150$ in^2

and $I = \int z^2\,dA = 15\int_{-5}^{+5} z^2\,dz = 1250$ in^4

Two programs were written to calculate the vertical deflection of ED dipole magnets for a variety of support spacings.

SAG2.FOR calculates the maximum deflection as a function of support separation. It shows that minimum deflection occurs when the support separation is 130" (55.3% of L) and that the deflection doubles if the separation is changed by ± 4" (± 1.7% of L). Minimum deflection occurs when the deflection at the ends equals the deflection at the center, and when the high point is nearly directly above the support. For each increment the supports are moved in or out, the high point moves approximately 6 increments. e.g. if the supports are spaced at 132", the high points are spaced at about 146".

SAG1.FOR calculates the deflection at various points for a given support spacing. Calculations for a support spacing of 130" and 140" is plotted along with measured results for a 142" support spacing. Data seems to imply that there is extra weight on the ends of the magnet.


Calculated for 130" supports
Calculated for 142" supports
AVERAGE DEFLECTION CURVE
142" SUPPORTS
21' LENGTH OF MAGNET
20
10
0
10
20

```
.TYPE SAG2.FOR
        REAL L
        W=8400.
        L=235.
        EI=7.87E9
        A=150.
        I=1250.
        F=64
        DO 40 J=1,20
        F=F+.1
        F2=F**2
        F3=F2*F
        F4=F3*F
        N=L/2+1
        X=-1
        YH=-1000.
        YL=1000.
        DO 10 I=1,N
        X=X+1
        X2=X**2
        X3=X2*X
        X4=X3*X
        IF(X.GT.F)GO TO 20
        Y=F*X2/4.+3.*I*X2/(2.*A*L)-L*X2/16.-X4/(24.*L)
        Y=W*Y/EI
        GO TO 30
20      Y=-F3/12.+F2*X/4.+3*I*X2/(2.*A*L)-L*X2/16.+X3/12.-X4/(24.*L
        Y=W*Y/EI
30      YF=F3/4.+3.*I*F2/(2.*A*L)-L*F2/16.-F4/(24.*L)
        YF=W*YF/EI
        Y=1000.*(Y-YF)
        IF(Y.GE.YH)YH=Y
        IF(Y.GE.YH)XH=X
        IF(Y.LE.YL)YL=Y
        IF(Y.LE.YL)XL=X
10      CONTINUE
        YD=YH-YL
        TYPE 100,F,YH,XH,YL,XL,YD
100     FORMAT(2X,F8.2,2(5X,F8.1,F8.0),2X,F8.1)
40      CONTINUE
        STOP
        END
```

```
TYPE SA61.FOR
              REAL L
              W=8400.
              L=235.
              EI=7.87E9
              A=150.
              I=1250.
              F=65.
              F2=F**2
              F3=F2*F
              F4=F3*F
              N=L/2+1
              X=-1
              DO 10 I=1,N
              X=X+1
              X2=X**2
              X3=X2*X
              X4=X3*X
              IF(X.GT.F)GO TO 20
              Y=F*X2/4.+3.*I*X2/(2.*A*L)-L*X2/16.-X4/(24.*L)
              Y=W*Y/EI
              GO TO 30
      20      Y=-F3/12.+F2*X/4.+3*I*X2/(2.*A*L)-L*X2/16.+X3/12.-X4/(24.*L)
              Y=W*Y/EI
      30      YF=F3/4.+3.*I*F2/(2.*A*L)-L*F2/16.-F4/(24.*L)
              YF=W*YF/EI
              Y=1000.*(Y-YF)
              TYPE 100,X,Y
      100     FORMAT(2X,F8.2,5X,F8.1)
      10      CONTINUE
              STOP
              END
```